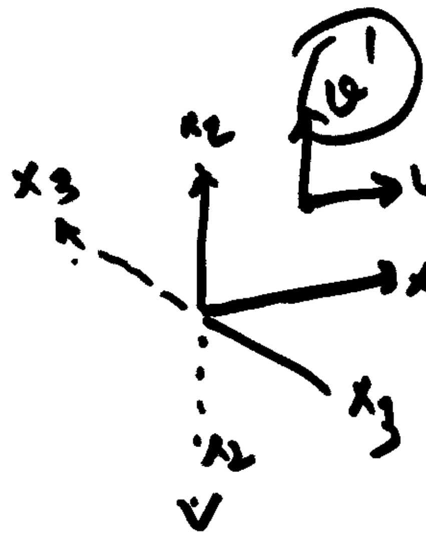
Prof. A.W. Date
Lec-22
Date-18-2-10

$\cdot\, u_x'(t+\Delta t)$

$\cdot\, u_x'(t)$

$$\overline{\hat{\phi}} = \frac{1}{t \to \infty} \int_0^{\infty} \hat{\phi}\, dt$$

$$\overline{\phi'} = \frac{1}{t_{max}} \int_0^{t_{max}} \phi'\, dt = 0$$



v' (-ve in the new system)

$$\left(\frac{\partial u_1'}{\partial x_1}\right)^2 = \frac{1}{2}\overline{\left(\frac{\partial u_1'}{\partial x_2}\right)^2}$$

$$S_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}$$

$$s_{ij}' = \frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}$$

s_{ij}

$$p\varepsilon = \mu\left[2\left(\frac{\partial u_1'}{\partial x_1}\right)^2 + 2\left(\frac{\partial u_2'}{\partial x_2}\right)^2 + 2\left(\frac{\partial u_3'}{\partial x_3}\right)^2 + \left(\frac{\partial u_1'}{\partial x_2} + \frac{\partial u_2'}{\partial x_1}\right)^2 + \left(\frac{\partial u_1'}{\partial x_3} + \frac{\partial u_3'}{\partial x_1}\right)^2 + \left(\frac{\partial u_2'}{\partial x_3} + \frac{\partial u_3'}{\partial x_2}\right)^2\right]$$

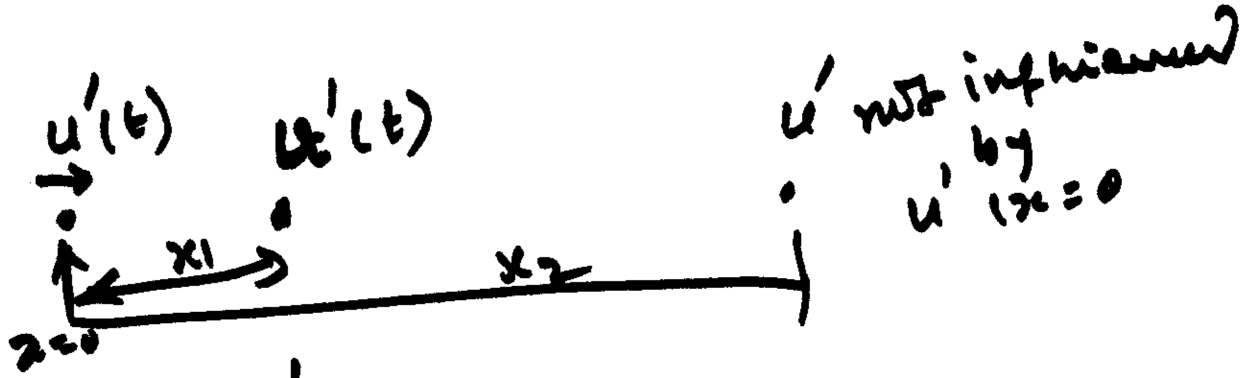
$$= \mu\left[6\left(\frac{\partial u_1'}{\partial x_1}\right)^2 + \left\{\left(\frac{\partial u_1'}{\partial x_2}\right)^2 + \left(\frac{\partial u_2'}{\partial x_1}\right)^2 + 2\left(\frac{\partial u_1'}{\partial x_2}\frac{\partial u_2'}{\partial x_1}\right)\right\}\right.$$

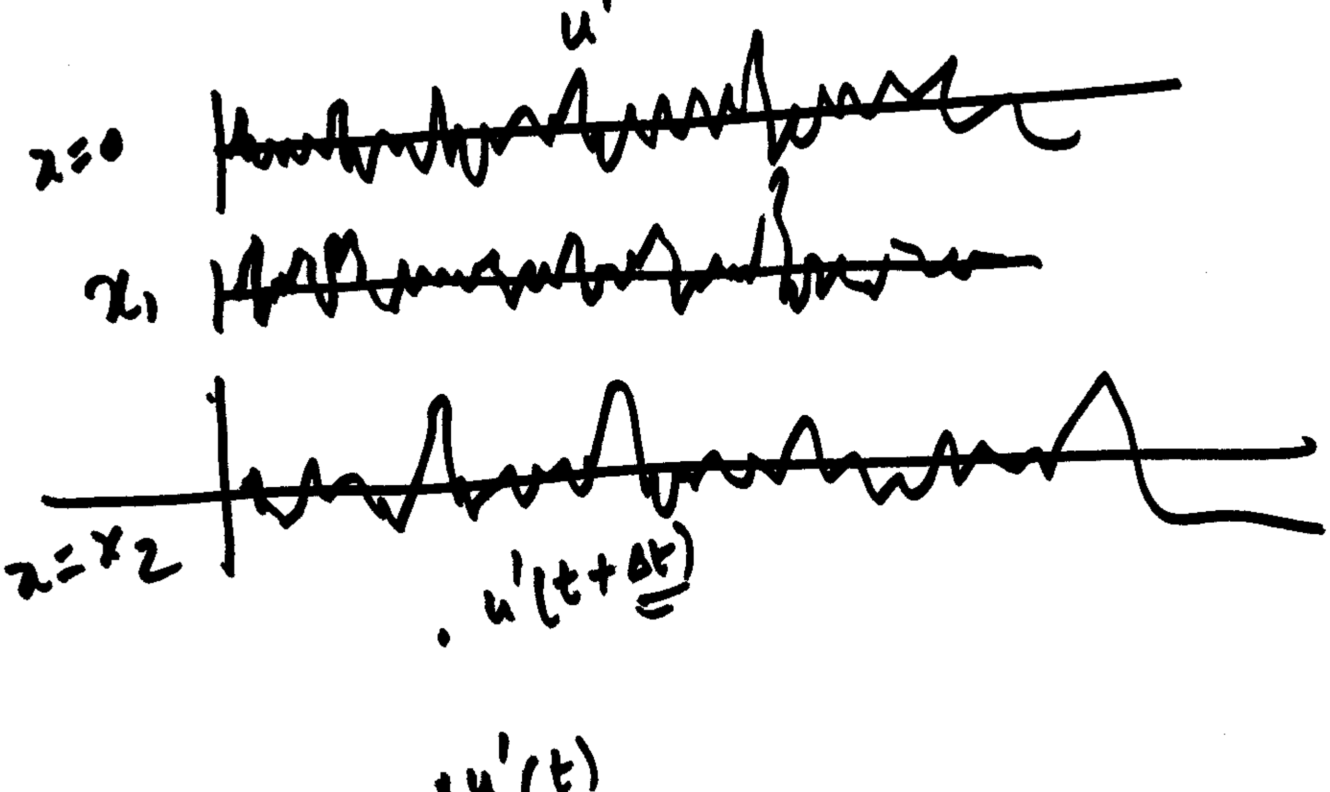
$$\left\{2\left(\frac{\partial u_1'}{\partial x_1}\right)^2 + 2\left(\frac{\partial u_1'}{\partial x_1}\right)^2 + 2\sqrt{2}\times\sqrt{2}\left(\frac{\partial u_1'}{\partial x_2}\right)\right\}$$

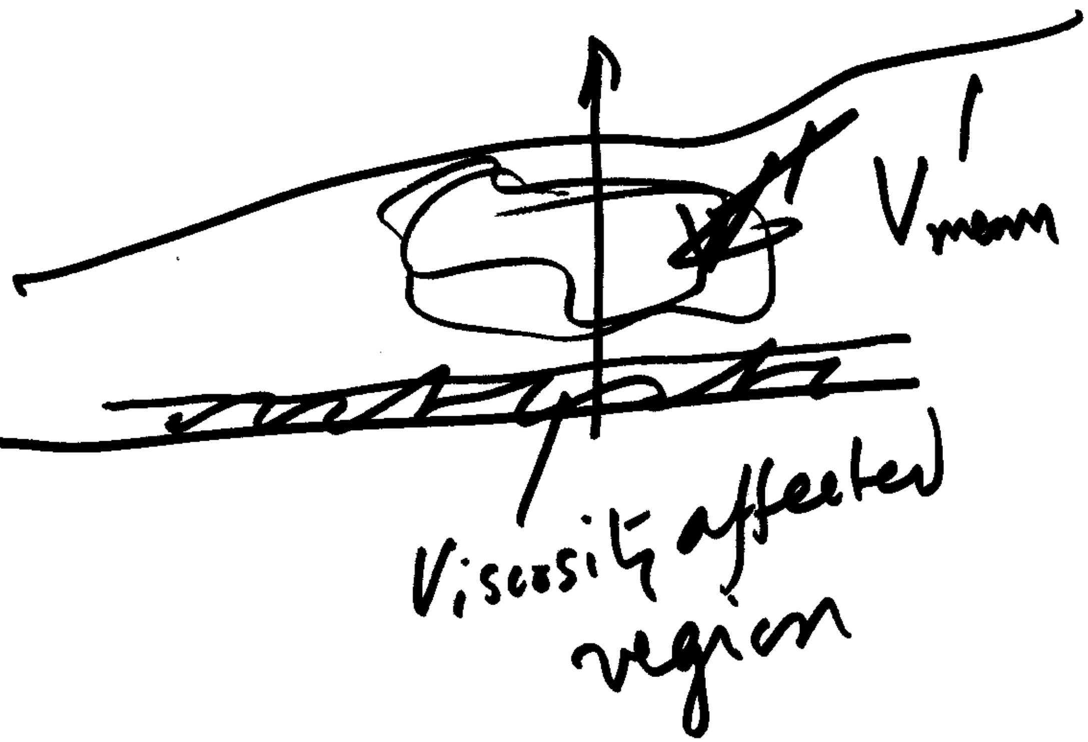
$$+ 8\left(\frac{\partial u_1'}{\partial x_1}\right)^2 \leftarrow$$

$$8\left(\frac{\partial u_1'}{\partial x_1}\right)^2$$

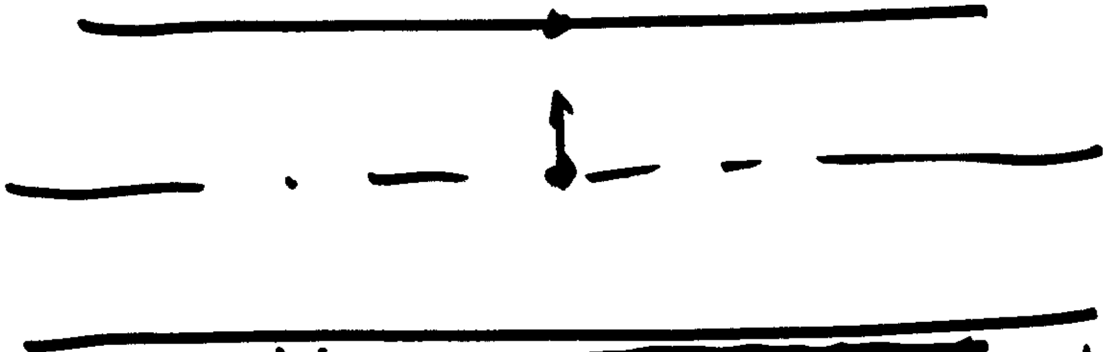
$$+ 8\left(\frac{\partial u_1'}{\partial x_1}\right)^2 = \mu\, 30\left(\frac{\partial u_1'}{\partial x_1}\right)^2$$


u'(t)
u'(t)
u' not influenced by u' |x=0
x1
x2
x=0


u'
x=0
x1
x=x2
u'(t+Δt)
u'(t)



$$\rho\varepsilon = \overline{\tau' \frac{\partial u_i'}{\partial x_j}}$$



$$\rho \int \frac{De'}{Dt}\, dV = \int \frac{\partial}{\partial y} \cancel{\left\{ \overline{v'\left(p' + \frac{u'^2+v'^2+w'^2}{2}\right)} \right\}} dV$$

$$+ \int \left(-\rho \overline{u'u'} \frac{\partial u}{\partial y}\right) \longrightarrow \text{Production}$$

$$+ \int \frac{\partial}{\partial y} \cancel{\left[\overline{u'\tau'_{yx}}\right]} - \int \left[\overline{\tau'_{yx} \frac{\partial v'}{\partial y}}\right] dV \qquad \tau'_{yx} = \mu \frac{\partial u'}{\partial y}$$

$$u_i \times \left[\rho \frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\tau_{ji} - \underbrace{\rho \overline{u_i' u_j'}}_{\text{turbulent stress.}}\right) \right]$$

$$\overline{E} = u_i^2/2$$

$$\tau_{ij}' = \mu\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)$$

$$\hat{E} = \frac{\hat{u}_i\hat{u}_i}{2} = \frac{\hat{u}_1^2+\hat{u}_2^2+\hat{u}_3^2}{2}$$

$$\hat{u}_i \times \rho \frac{D\hat{u}_i}{Dt} = -\frac{\partial \hat{p}}{\partial x_i} + \frac{\partial(\hat{\tau}_{ji})}{\partial x_j} \times \hat{u}_i$$

$$\rho \frac{D\,\hat{u}_i\hat{u}_i/2}{Dt} = \frac{\rho D\hat{E}}{Dt} = -\hat{u}_i\frac{\partial\hat{p}}{\partial x_i} + \hat{u}_i\frac{\partial\hat{\tau}_{ji}}{\partial x_j}$$

$$\therefore \rho \cdot \frac{D\hat{E}}{Dt} = -\frac{\partial(\hat{p}\hat{u})}{\partial x_i} + \bar{p}\cancel{\frac{\partial\hat{u}_i}{\partial x_i}} + \frac{\partial}{\partial x_j}(\hat{u}_i\hat{\tau}_{ji}) - \underbrace{\hat{\tau}_{ji}\frac{\partial\hat{u}_i}{\partial x_j}}_{\mu\Phi_V}$$